

Inference at \* 1  
of proof for Lemma exists\_functionality\_wrt\_iff:

1.  $S : \text{Type}$
2.  $T : \text{Type}$
3.  $P : S \rightarrow \mathbb{P}$
4.  $Q : S \rightarrow \mathbb{P}$
5.  $S = T$
6.  $\forall x:S. P(x) \iff Q(x)$
7.  $\exists x:S. P(x)$

$\vdash \exists y:T. Q(y)$

by ((((((D 7

CollapseTHEN (With  $x$  (D 0))))))

CollapseTHENM (HypBackchain))))

CollapseTHEN ((Auto\_aux (first\_nat 1:n) ((first\_nat 1:n),(first\_nat 3:n)) (first\_tok :t  
) inil\_term))))

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